SAF SCHOOLS ALLIANCE

CALCULATION POLICY

Stotfold • Arlesey • Fairfield Park

This calculation policy is part of the work of a wider collaboration to offer the best transition for pupils between Lower and Middle

Final Copy agreed by SAF Group. January 2022

Schools with the ultimate goal of enhanced outcomes at the end of the Primary Phase (Yr6). It has been written and developed with involvement from all the math's subject leaders and relevant colleagues from the six schools in the SAF group: Etonbury Academy, Pix Brook Academy, Fairfield Park, Gothic Mede Academy, Roecroft Lower School and St Mary's Church of England Academy Stotfold. It has been developed from a number of national sources of best practice and reflects the current higher expectations of maths in the Primary Phase.

The policy sets out, year group by year group, the progression of calculation methods (addition, subtraction, multiplication and division) expected for the Primary Phase.

Understanding the document:

While this document has been developed for use by teachers in school, we hope that parents and carers will, with a few explanations, find it helpful in supporting maths activities with their child at home. The methods in this document are not exhaustive and from time to time teachers may use alternative methods which are better suited to your child's understanding and development at a given point in time.

The headings for each stage of development are:

Concrete – here the expectation is that a student would use equipment or manipulative techniques e.g. counters, blocks, compare bears, Numicom to develop their understanding of a concept.

This leads to …

Pictorial – these strategies begin to move away from sole use of equipment to develop the written calculation. This could include jottings or illustrations of the concept being taught. It might involve words or images which reflect the student's working through a particular calculation. It could also include informal methods that assist in 'proving' a concept to the student.

This leads to ….

Abstract – this shows the formal method of representing a calculation as agreed by all the schools involved. It is the average, end of year expectation. Some children may access these methods before the end of a given year while others will continue to develop their understanding through either Pictorial or Concrete methods. The 'Abstract' methods will probably look most familiar to how we as parents and carers view or represent mathematical calculations.

If your child has not yet developed a clear understanding of the 'Abstract' for their year group, you may find that they are using the 'Abstract' method from the previous year as a bridge to moving from the informal to expected formal method.

Objectives, strategies and vocabulary:

This indicates the method and strategy being described. There is a list of the key vocabulary to be used with this method.

Concrete:

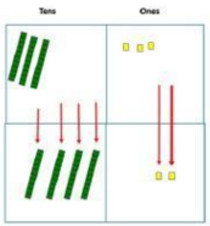
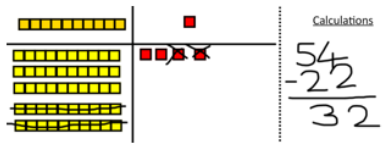
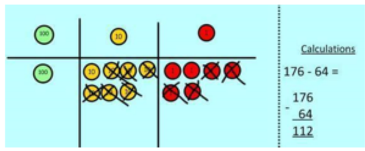
This section gives suggestions on manipulatives, equipment and techniques which colleagues may wish to use to develop understanding. This is not a definitive list.

Pictorial/Jottings:

When the children are ready they can move onto this section. The children will move away from using manipulatives and begin to use pictures/ jottings to help them calculate their answers.

Headings:

Each page indicates the year group and strategy. For ease of use each operation is a different colour.

	Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract
Year 3 Subtraction	Column method without regrouping subtract, take (away), minus leave, how many are left/left over? one less, two less… ten less… one hundred less how many fewer is… than …? how much less is…? difference between half, halve =, equals, sign, is the same as Exchanging Carrying partitioning	 Use Base 10 to make the bigger number then take the smaller number away. 75 - 42 33	Draw the Base 10 or place value counters alongside the written calculation to help to show working  	This will lead to a clear written column subtraction. 599 - 234 365

Abstract:

This is the agreed formal method of calculation; it is also the average end of year expectation.

SAF Calculation Policy Overview

EYFS

The objective for those working in Early Years is to ensure that all children develop firm mathematical foundations in a way that is engaging, and appropriate for their age. There are six key areas of early mathematics learning, which collectively provide a platform for everything children will encounter as they progress through their maths learning at primary school, and beyond:

- **Cardinality and Counting**

  Understanding that the cardinal value of a number refers to the quantity, or 'howmanyness' of things it represents. To help children to develop a strong understanding of cardinality, they are taught to **subitise.**

  - **Subitising**

    Children learn to recognise the number of objects in a group (up to five) without counting, through 'hidden objects' games and games using dice and dominoes. It is important that children develop strong images of familiar patterns, such as those on dice, but also that they see small numbers arranged in unfamiliar patterns.

- **Comparison**

  Understanding that comparing numbers involves knowing which numbers are worth more or less than each other

- **Composition**

  Understanding that one number can be made up from (composed from) two or more smaller numbers

- **Pattern**

  Looking for and finding patterns helps children notice and understand mathematical relationships

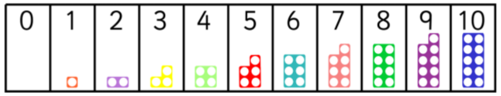
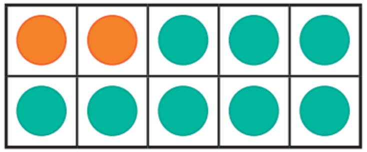
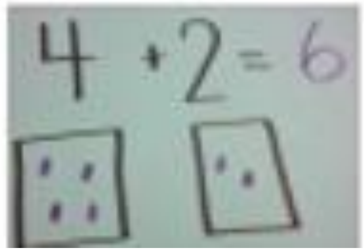
- **Shape and Space**

  Understanding what happens when shapes move, or combine with other shapes, helps develop wider mathematical thinking

- **Measures**
- Comparing different aspects such as length, weight and volume, as a preliminary to using units to compare later

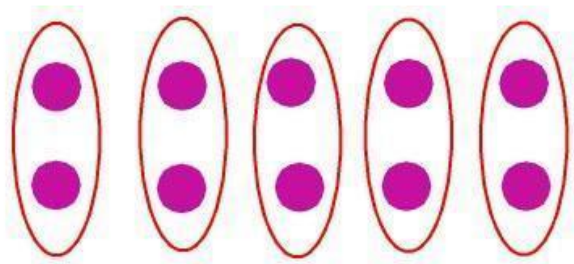
More information about these six key areas can be found at: https://www.ncetm.org.uk/in-the-classroom/early-years/

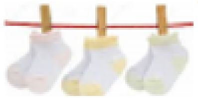
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Partitioning method.	Start with: Column method – no regrouping. Moving to: Column method-regrouping. (up to 3 digits)	Column method-regrouping. (up to 4 digits)	Column method-regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method-regrouping. (Decimals- with different amounts of decimal places)
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back and find the difference using number lines. Part whole model Make 10	Start with: Column method-no regrouping Move to: Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method with regrouping. (Decimals- with different amounts of decimal places)
Multiplication	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication	Counting in multiples Repeated addition Arrays- showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (multi digit up to 4 digits by a 2 digit number)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder	Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number- interpret remainders as whole numbers, fractions or round)

Objectives, Strategies & Vocabulary		Concrete	Pictorial/Jottings	Abstract
EYFS Addition	**EYFS Method** Plus Estimate Add More And Total Make Altogether Double One more, two more, ten more How many More make..? How many more is.. than..? Same as	If available, Numicon shapes are introduced straight away and can be used to: • Identify 1 more/less • Combine pieces to add • Find number bonds • Add without counting  Children can begin to combine groups of objects using concrete apparatus: Five and tens frames are used to support with addition, with the composition of number and with number bonds to 5 and 10:   Children solve simple problems using fingers Number tracks can be introduced to count up on and to find one more: 	Children may make a record in pictures, words or symbols of their addition activities. 	Children are encouraged to read number sentences aloud in different ways, usually using numbers between 1 and 20 e.g. 'three add two equals 5", "5 is <u>equal</u> to three and two" "five is <u>the same as</u> three and two", 

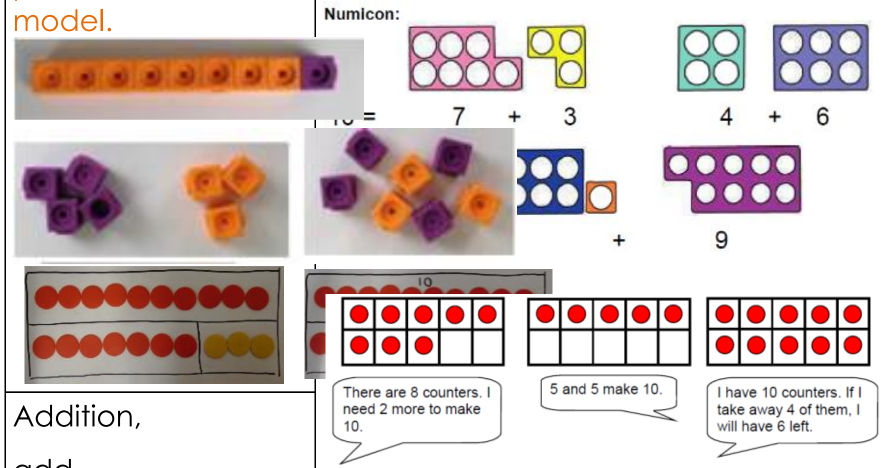
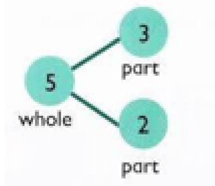
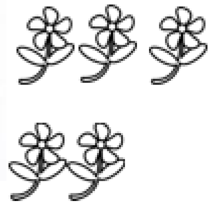
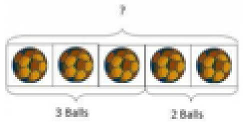
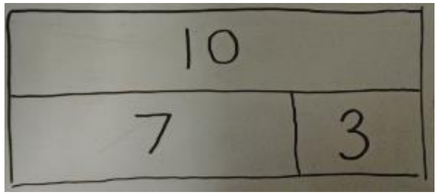
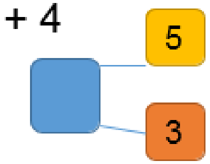
		What is 1 more than 4? 1 more than 13? Number lines can be used alongside number tracks and practical apparatus to solve addition calculations and word problems. Children will need opportunities to look at and talk about different models and images as they move between representations.		

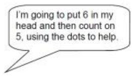
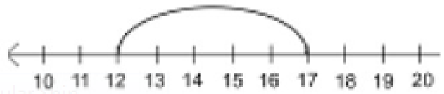
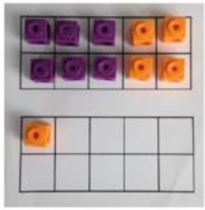
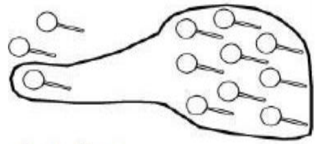
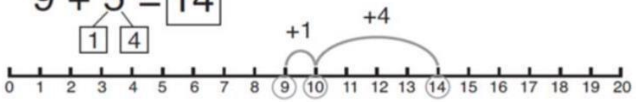
Objectives, Strategies & Vocabulary		Concrete	Pictorial/Jottings	Abstract
EYFS Subtraction	Take (away) Subtract Estimate Leave How many are left/ left over? How many have gone? One less, two less, ten less How many fewer is…than..? Difference between The same as jumping back	Children begin with mostly pictorial representations or real contexts.  Concrete apparatus is used to relate subtraction to taking away and counting how many objects are left.  Concrete apparatus models the subtraction of 2 objects from a set of 6. Solve simple problems using fingers. Number tracks can be introduced to count back and to find one less: What is 1 less than 9? 1 less than 20?  Number lines can then be used alongside number tracks and practical apparatus to solve subtraction calculations and word problems. Children count back showing hops back on the number back. Children will need opportunities to look at and talk about different models and images as they move between representations.	Construct number sentences using cards to go with practical activities. Children make a record in pictures, words or symbols of subtraction activities. 	Children are encouraged to read sentences aloud in different ways "five subtract one leaves four", "four is equal to five subtract one", "four is the same as five subtract one"

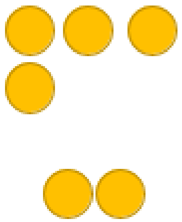
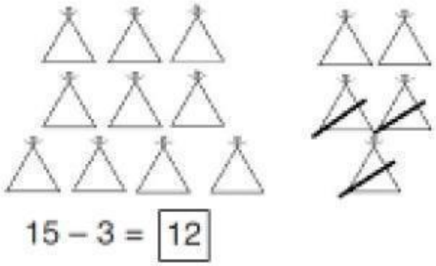
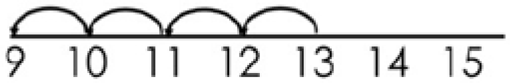
	Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	
EYFS Multiplication	Lots of Groups of Times Once, twice, three times..ten times …times as (big, long, wide..and so on) Repeated addition Double Estimate Add again and again	If available, Numicon is used to visualise the repeated adding of the same number. Real life contexts and use of practical equipment to count in repeated groups of the same size: How many wheels are there altogether? How much money do I have? Count in twos, fives, tens both aloud and with objects.	Children begin with mostly pictorial representations: How many groups of 2 are there? 2+2+2+2+2, so 5 groups of 2.  	Children are given multiplication problems set in a real life context. Children are encouraged to visualise the problem. How many fingers on two hands? How many sides on three triangles? How many legs on four ducks?

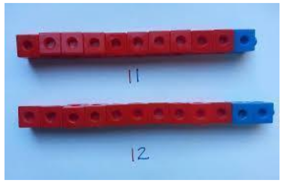
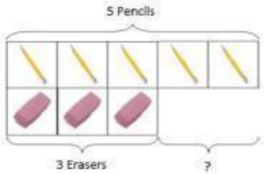
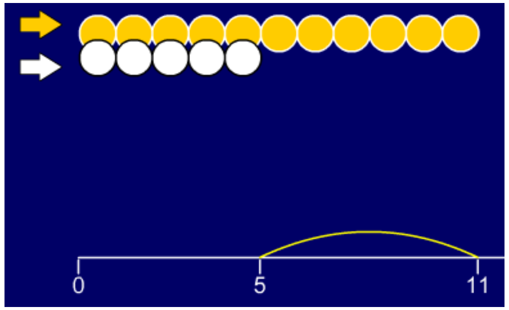
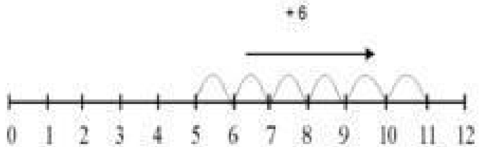
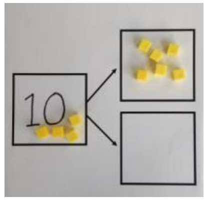
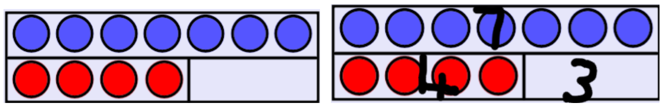
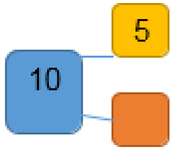
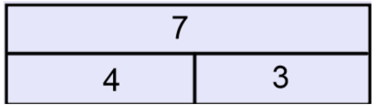
	Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract
EYFS Division	Halve Share Share, share equally One each, two each, three each…. Groups in pairs, threes… Tens Equal groups of Divide Divided by Divided into Left Left over	Children begin with mostly pictorial representations linked to real life contexts. Mum has 6 socks. She grouped them into pairs – how many pairs did she make? How many socks did she have altogether? Sharing model: I have 10 sweets. I want to share them with my friend. How many will we have each? Although not explicit in the development matters document, the sharing model is a useful way of introducing young children to fractions and calculating with fractions. Setting the problems in a real life context and solving them with concrete apparatus will support children's understanding.  	Children make a record of division in words, pictures or symbols of their division activities. 	

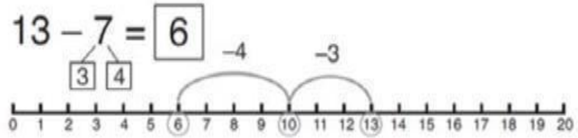
Year 1 Addition

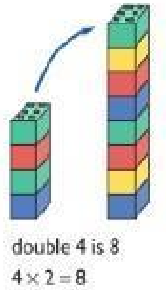
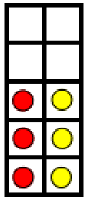
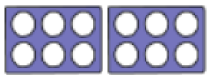
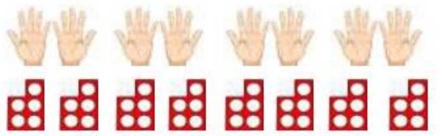
Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract
Combining two parts to make a whole: part-whole model. Addition, add, forwards, put together, more than, total, altogether, distance between, counting on, equals = same as, counting on, pattern.	 Numicon and ten-frame resources can provide the first step into understanding 2-digit numbers. It will be useful at this point to introduce children to Base 10 resources and use then to partition 'teen' numbers into tens and ones.	 Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract. Through all of the stages, children should start to recognise the **relationship** between **addition** and **subtraction**.

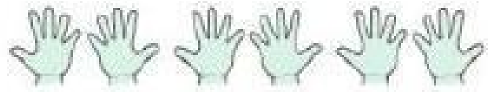
	Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract
Year 1 Addition	Starting at the bigger number and counting on.	 5 and 1 more is **6** 1, 2, 3, 4, 5.........**6**  5 and 2 more is **7** 1, 2, 3, 4, 5.........6, **7**  Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.  +  = 11  Eventually, as children become more competent they will be able to hold the **biggest number** in their head and then **count on**- perhaps using their fingers- from there.	12 + 5 = 17  Start at the larger number on the number line and count on in **ones** or in **one jump** to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to you answer.
	Regrouping to make 10.	  Start with the bigger number and use the smaller number to make 10. 6 + 5 = 11	 **3 + 9 =** Use pictures or a number line. Regroup or partition the smaller number to make 10. 9 + 5 = 14 1 4 	7 + 4 = 11 If I'm at 7, how many more do I need to make 10? How many more do I add on now?

	Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract
Year 1 Subtraction	Taking away ones -, subtract, subtraction, take away, minus, less than, most, least, distance between, difference between, equals =, same as, digit.	Use physical objects, counters, cubes etc. to show how objects can be taken away. 6 – 2 = 4 	Cross out drawn objects to show what has been taken away. 	8 – 2 = 6 18 – 3 = 15
	Counting back	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

Objectives, Strategies & Vocabulary		Concrete	Pictorial/Jottings	Abstract
Year 1 Subtraction	Find the difference	Compare amounts and objects to find the difference. Use cubes to build towers or make bars to find the difference.   Use basic bar models with items to find the difference.	11 = 5 =  ITP Difference  Count on to find the difference.	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the numbers of sandwiches.
	Part Part Whole Model	**Link to addition** - use the part whole model to help explain the inverse between addition and subtraction.  If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6=	Use a pictorial representation of objects to show the part whole model. 	 Move to using numbers in the part whole model. 

	Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract
Year 1 Subtraction	Make 10	14 – 5 =  Make 14 on the ten frame. Take away the 4 first to make 10 and then takeaway 1 more so you have taken away 5. You are left with the answer of 9.	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8 =7 How many do we need to take off to reach the next 10? How many do we have left to take off?

<table>
<tr><th colspan="2">Objectives, Strategies & Vocabulary</th><th>Concrete</th><th>Pictorial/Jottings</th><th>Abstract</th></tr>
<tr><td rowspan="3">Year 1 Multiplication</td><td>Doubling</td><td rowspan="2">Use practical resources to show how to double a number.



6 + 6 = 6 x 2 = 12</td><td rowspan="2">Draw pictures to show how to double a number.
Double 4 is 8
</td><td rowspan="2">Know that doubling a number is the same as two lots of the same number.</td></tr>
<tr><td>Ones, groups of, lots of, doubling, repeated addition, groups of, lots of, times, columns, rows, longer, bigger, higher, times as (big, long, wide… etc.), array.</td></tr>
<tr><td>Counting in multiples</td><td>
2, 4, ..

5, 10, 15, 20, 25, 30, 35, 40..

10, 20, 30,
Count in multiples supported by concrete objects in equal groups.</td><td>

Use a number line or pictures to continue to support in counting in multiples.</td><td>Count in multiples of a number aloud.

Write sequences with multiples of numbers.

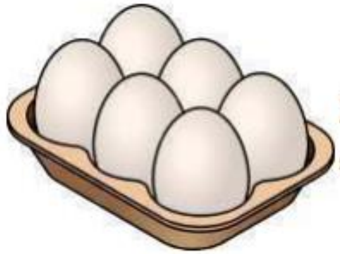
0, 2, 4, 6, 8, 10 …

0, 5, 10, 15, 20, 25, 30 …

0, 10, 20, 30, 40, 50 …</td></tr>
</table>

Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings
Year 1 Multiplication — Arrays (with support)		

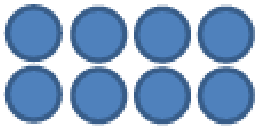
Arrays will be used to help children visualise and understand multiplication and division.



2 x 3 is the same as 3 x 2

These everyday items, arranged in rows and columns, highlight an important multiplication fact to the children: that multiplication can be done in any order (**commutative**).

Here is an array.



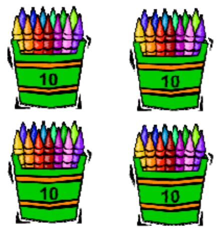
There are 2 rows of 4 counters.
There are 8 altogether.

Can you arrange the counters in a different way so there are a different number of equal rows?
Can you do the same with 15 counters?

How many questions can you write about the gingerbread men?
e.g. How many buttons?

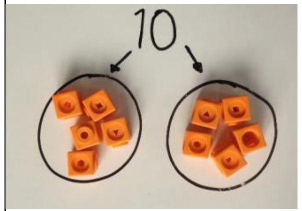
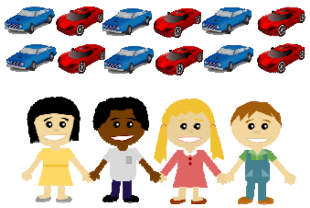
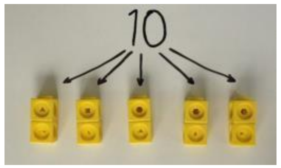
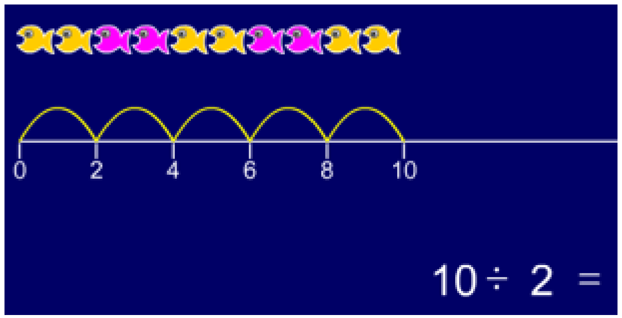
Harry has 3 friends.
Each friend gives him 5 sweets.
How many sweets does he have altogether?

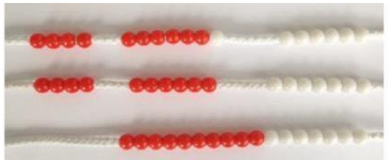
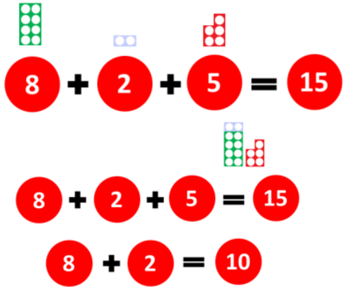
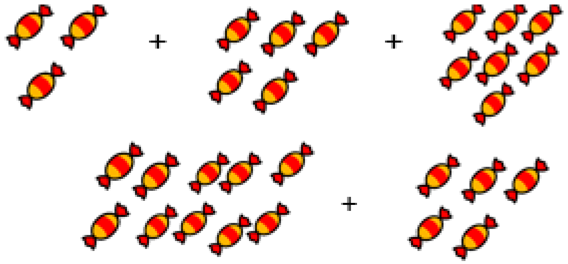
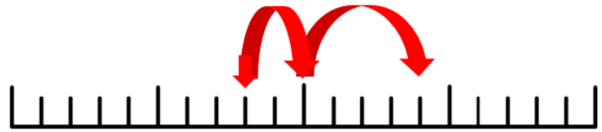
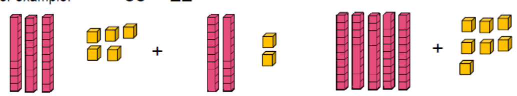
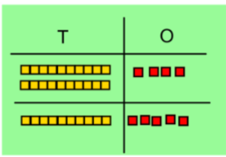
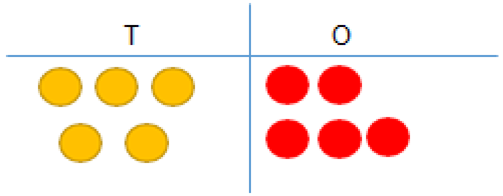
Make a word problem/story about the equal groups in the pictures

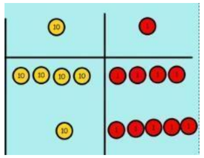
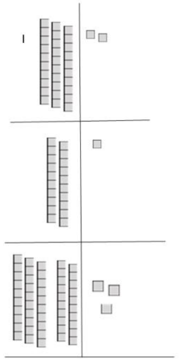
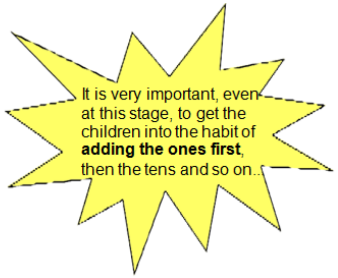


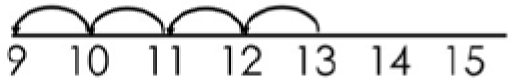
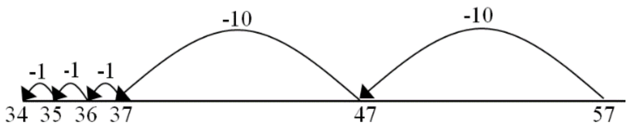
There are 10 crayons in each box. How many are there altogether?

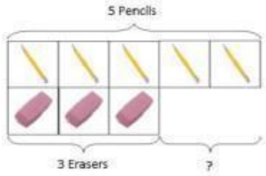
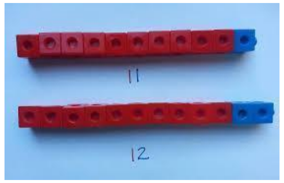
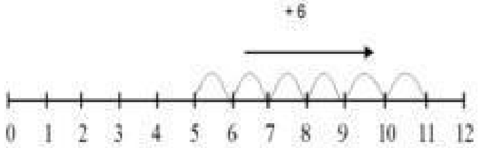
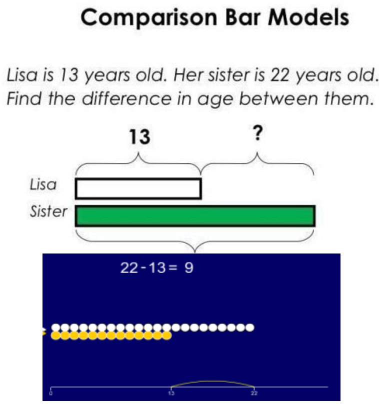
10 + 10 + 10 + 10 = 40 crayons
There 4 groups of 10
10 x 4 = 40
10 multiplied by 4 is 40

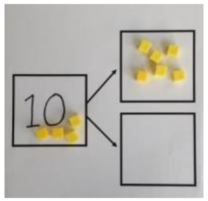
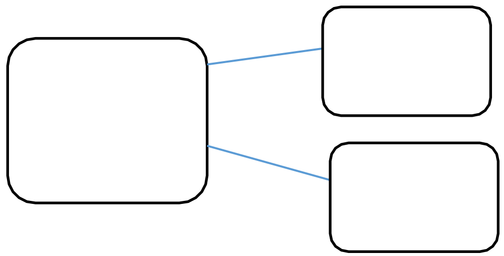
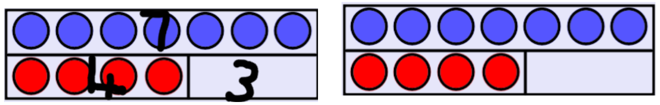
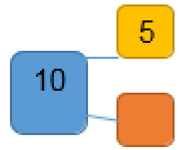
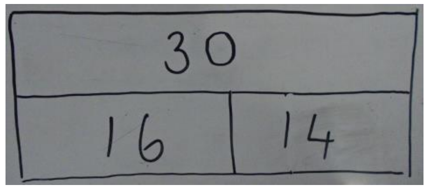
	Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract
Year 1 Division	Sharing objects into groups Share, share equally, one each, two each, group, groups of, lots of, arrays.	   I have 10 cubes. Can you share them equally in 2 groups?  Four children share 12 toy cards equally. How many do they get each?	Children use pictures or shapes to share quantities.  $\boxed{8 \div 2 = 4}$	Share 9 buns between three people. $9 \div 3 = 3$
	Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.   		

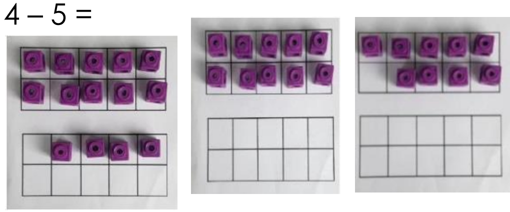
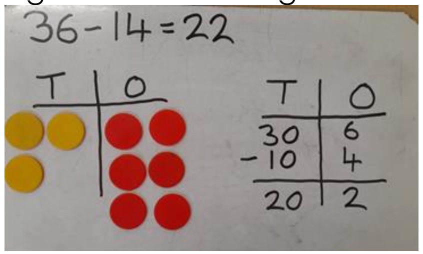
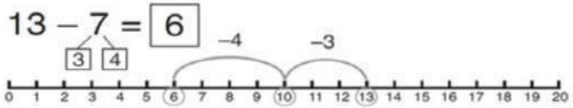
	Objectives, Strategies & vocabulary	Concrete	Pictorial/Jottings	Abstract
Year 2 Addition	Adding 3 single digits. +, add, addition, more, plus, make, sum, total, altogether, how many more to make…? how many more is… than…? =, is the same as, tens, ones, partition, more than, one more, two more, ten more, one hundred more	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7.   	 Add together three groups of objects. Draw a picture to recombine the groups to make 10. 8 + 4 + 2 =  	4 + 7 + 6 = 10 + 7 10 = 17 Combine the two numbers that make 10 and then add on the remainder.
	Column method- no regrouping	Initially, the children may use base 10 equipment to **partition** numbers in their tens and ones and then add them separately. For example: 35 + 22  30 + 5 + 20 + 2 = 50 + 7 24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.  	After practically using the base 10 blocks and place value counters, children can draw the counters or base 10 to help them to solve additions.  	24 + 15 = 4 + 5 = 9 20 + 10 = 30 30 + 9 = 39 When children are secure using the concrete method using concrete manipulatives and pictorially, they can move on to the abstract method: 32 +21 53

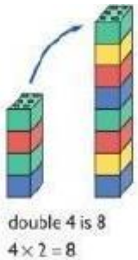
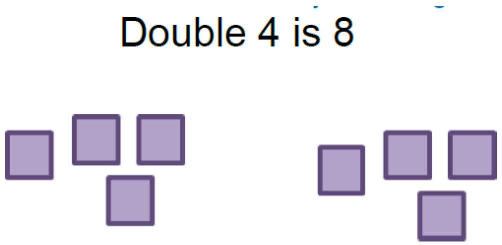
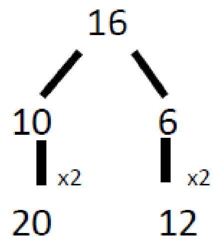
		44 + 15 = 	 	

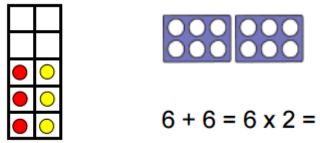
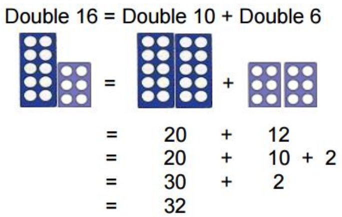
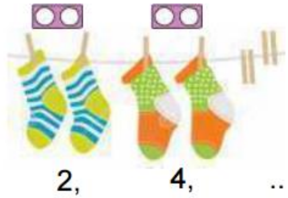
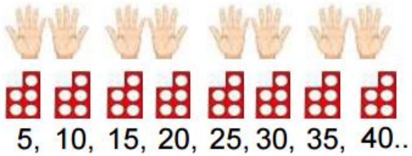
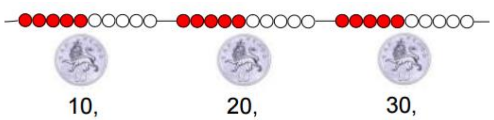
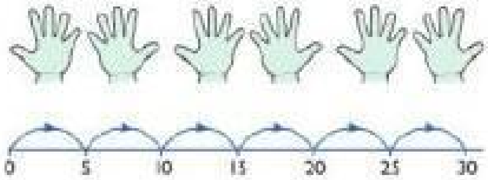
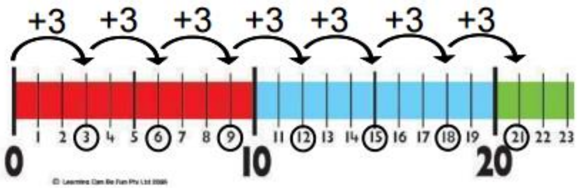
Objectives, Strategies & vocabulary		Concrete	Pictorial/Jottings	Abstract
Year 2 Subtraction	Counting back -, subtraction, subtract, take away, difference, difference between, minus, less than, one less, two less, ten less, one hundred less	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4  Use counters and move them away from the group as you take them away counting backwards as you go. 	Count back on a number line or number track  Start at the bigger number and count back the smaller number showing the jumps on the number line.  This can progress all the way to counting back using two 2 digit numbers.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help. 57 – 23 = Put 57 in your head, count back two steps of 10 and then three 1s.

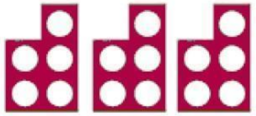
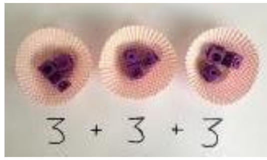
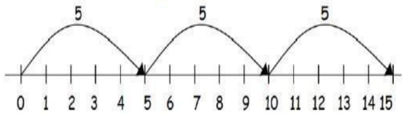
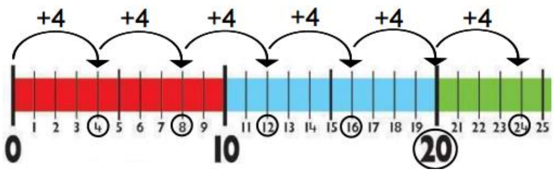
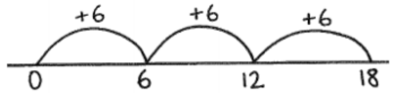
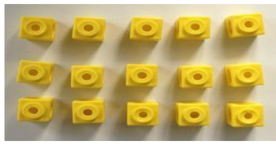
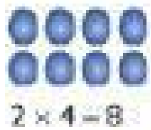
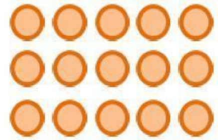
	Find the difference	Compare amounts and objects to find the difference.  Use cubes to build towers or make bars to find the difference.  Use basic bar models with items to find the difference.	Count on to find the difference.  Draw bars to find the difference between 2 numbers.  	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the numbers of sandwiches. $56 - \boxed{} = 51$

Objectives, Strategies & vocabulary		Concrete	Pictorial/Jottings	Abstract
Year 2 Subtraction	Part Whole Model	**Link to addition- use the part whole model to help explain the inverse between addition and subtraction.**  If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6=	Use a pictorial representation of objects to show the part whole model.   	  Move to using numbers in the part whole model and bar model.

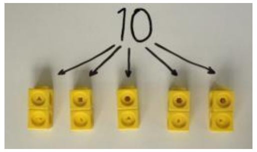
	Make 10	14 – 5 =  Make 14 on the 10 frame. Take away the 4 first to make 10 and then takeaway 1 more so you have taken away 5. You are left with the answer of 9. Show how you partition numbers to subtract. Again make the larger number first. 	 Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8 = 8 How many do we need to take off to reach the next 10? How many do we have left to take off?

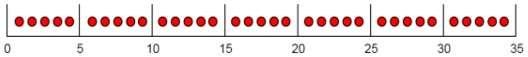
Objectives, Strategies & Vocabulary		Concrete	Pictorial/Jottings	Abstract
Year 2	Doubling x, multiple, multiplication array, multiplication table/facts, groups of, lots of, times, columns, rows, group in pairs, 2s, 3s, 5, 10s etc.	Use practical resources to show how to double a number. 	Draw pictures to show how to double a number. 	6 + 6 = 6 x 2 = 12  Partition a number and then

		 6 + 6 = 6 x 2 = 12 As well as knowing doubles up to 10 + 10, children should use these known facts to double 2-digit numbers.  		double each part before recombining it back together.
	Counting in multiples	    Count in multiples supported by concrete objects in equal groups.	 Use a number line or pictures to continue to support in counting in multiples.  	Count in multiples of a number aloud (forwards and backwards). Write sequences with multiples of numbers. 2, 4, 6, 8, 10 … 3, 6, 9, 12, 15 … 5, 10, 15, 20, 25, 30 … 10, 20, 30, 40, 50 … Knowledge of the 2 times table will enable children to count in 20s, 5s in 50s etc.

	Objectives, Strategies & vocabulary	Concrete	Pictorial/Jottings	Abstract
Year 2 Multiplication	Repeated Addition	 5 + 5 + 5  	There are 3 plates. Each plate has 2 star biscuits on. How many biscuits are there?  2 add 2 add 2 equals 6  5 + 5 + 5 = 15 6 x 4 = ? 6 lots of 4 4 + 4 + 4 + 4 + 4 + 4 = 24 6 x 4 = 24 	Write addition sentences to describe objects and pictures.  2 + 2 + 2 + 2 + 2 = 10 Some children may be able to use a blank number line to record their mental processes.  3 x 6 = ? 3 lots of 6 6 + 6 + 6 = 18 3 x 6 = 18
	Arrays – showing commutative multiplication	Arrays will be used to help children visualise and understand multiplication and division.  3 x 4 is the same as 4 x 3 These everyday items, arranged in rows and columns, highlight an important multiplication fact to the children: that multiplication can be done in any order (**commutative**).   Create arrays using counters/cubes to show multiplication sentences.	 4 × 2 = 8 2 × 4 = 8  2 × 4 = 8 4 × 2 = 8 Draw arrays in different rotations to find **commutative** multiplication sentences.	Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15

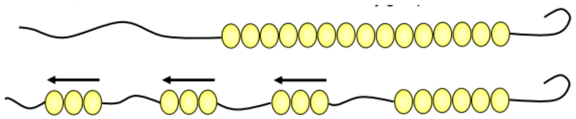
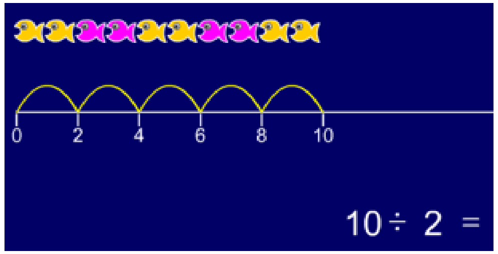
<table>
<tr><th colspan="2">Objectives, Strategies & Vocabulary</th><th>Concrete</th><th>Pictorial/Jottings</th><th>Abstract</th></tr>
<tr><td rowspan="3">Year 2 Division</td><td>Division as grouping</td><td rowspan="2">Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.





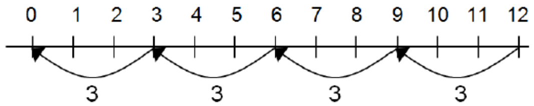
When grouping, you count the number of groups you have made.

For instance, 15 ÷ 3 = 5 can be viewed as ‘How many groups of 3 are there in 15?’

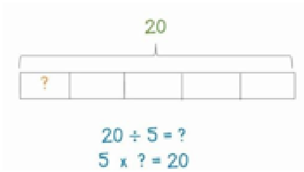
</td><td rowspan="2">

ITP Grouping

Use a number line to show jumps in groups. The number of jumps equals the number of groups.



Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.


</td><td rowspan="2">28 ÷ 7 = 4

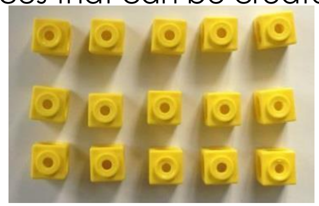
Divide 28 into 7 groups. How many are in each group?

Miss Smith needs **30** apples for her class.

There are **5** apples in each bag.

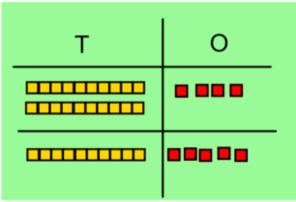
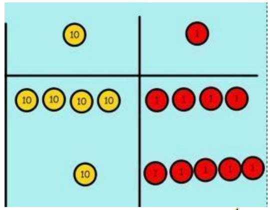
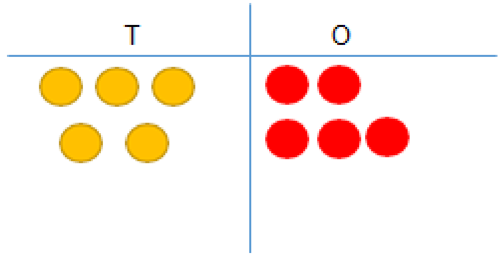
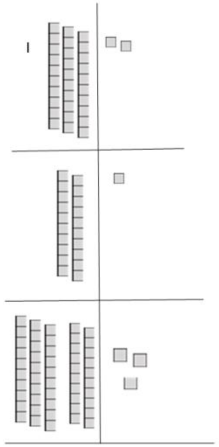
How many **bags** of apples does Miss Smith need altogether?

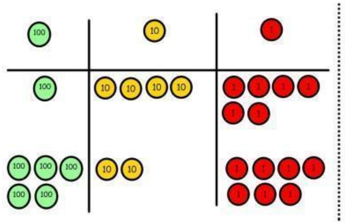
bags</td></tr>
<tr><td>÷, divide,
divided by,
divided into,
shared into,
columns, rows,
groups of</td></tr>
<tr><td>Division within arrays</td><td>Link division to multiplication by creating an array and thinking about the number sentences that can be created.

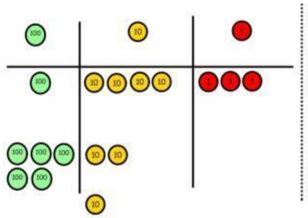


E.g. 15 ÷ 3 = 5   5 x 3 = 15
15 ÷ 5 = 3   3 x 5 = 15</td><td>Draw an array and use lines to split the array into groups to make multiplication and division sentences.</td><td>Find the inverse of multiplication and division sentences by creating four linking number sentences.

7 x 4 = 28
4 x 7 = 28
28 ÷ 7 = 4
28 ÷ 4 = 7</td></tr>
</table>

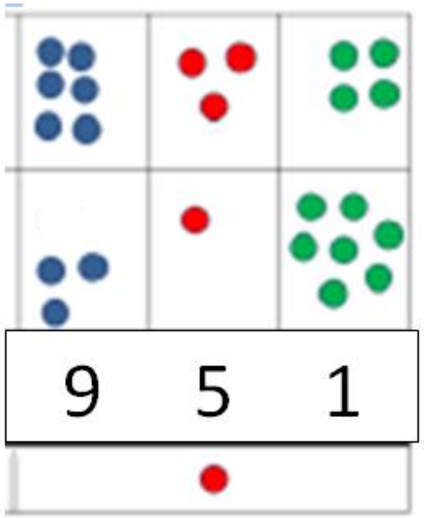
Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract
Year 3 Addition Column method - no regrouping +, add, addition, more, plus make, sum, total altogether score double, near double one more, two more… ten more… one hundred more how many more to make …? how many more is… than …? how much more is…? carry	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.   It is very important, even at this stage, to get the children into the habit of **adding the ones first**, then the tens and so on.	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions.   32 +21 53	This will lead to a clear, written column addition: 275 + 122 397

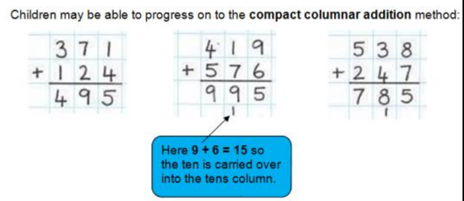
<table>
<tr><th colspan="2">Objectives, Strategies & Vocabulary</th><th>Concrete</th><th>Pictorial/Jottings</th><th>Abstract</th></tr>
<tr>
<td>Year 3 Addition</td>
<td>Column method - regrouping
(Up to 3 digits)</td>
<td>Make both numbers on a place value grid.

146
+ 527

Add up the ones and exchange 10 ones for one 10.

146
+ 527
=673

Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

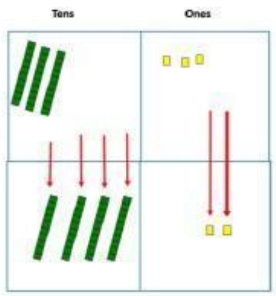
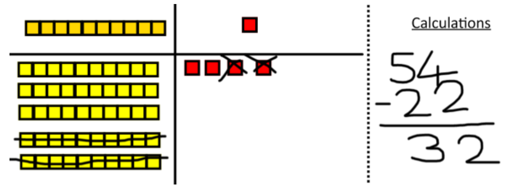
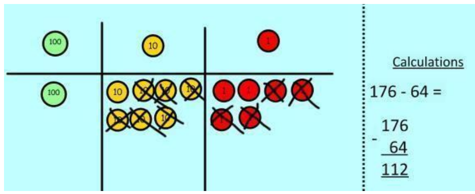
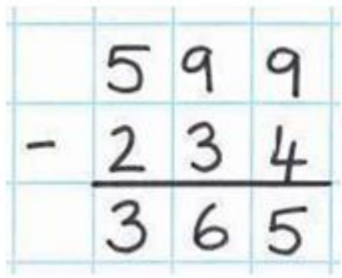
As children move on to decimals, money and decimal place value counters can be used to support learning.</td>
<td>Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding.


634
+ 317</td>
<td>

As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

A common misconception:
Pupils sometimes begin adding with the left hand column first.
Not understanding the concept of a ‘carry’ when a number totals more than ten, hundred etc.
e.g.
101
1910

+
72.8
+ 54.6
127.4
1 1</td>
</tr>
</table>

Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract
Year 3 Subtraction Column method without regrouping subtract, take (away), minus leave, how many are left/left over? one less, two less… ten less… one hundred less how many fewer is… than …? how much less is…? difference between half, halve =, equals, sign, is the same as Exchanging Carrying partitioning	 Use Base 10 to make the bigger number then take the smaller number away. 75 - 42 33	Draw the Base 10 or place value counters alongside the written calculation to help to show working  	This will lead to a clear written column subtraction. 

Final Copy agreed by SAF Group. January 2022

Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract
Year 3 Subtraction Column method with regrouping (up to 3 digits)	See below	See below	See below

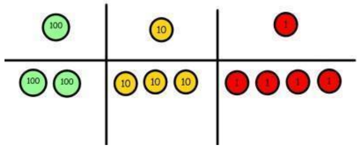
Year 3 Subtraction

Column method with regrouping (up to 3 digits)

Concrete

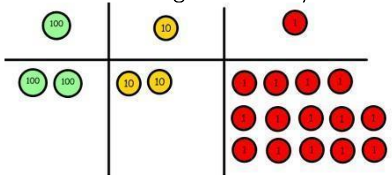
Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

Make the larger number with the place value counters



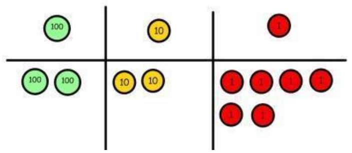
Calculations
234
- 88

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



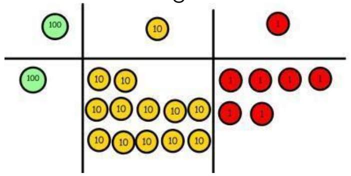
Calculations
234
- 88

Now I can subtract my ones.



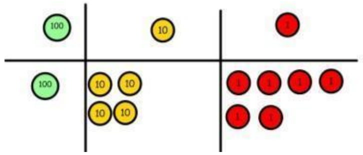
Calculations
234
- 88

Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Calculations
234
- 88

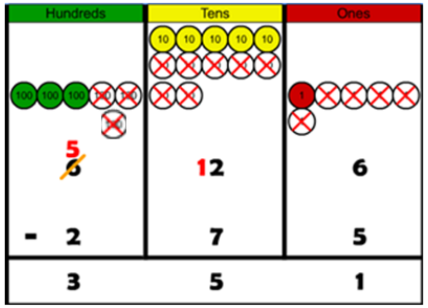
Now I can take away eight tens and complete my subtraction



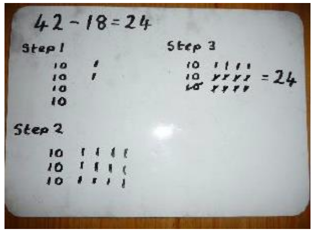
Calculations
$^{1}\cancel{2}\,^{13}\cancel{3}4$
- 88
146

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

Pictorial/Jottings



Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.

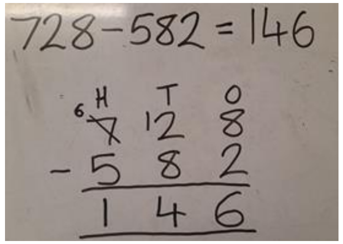


When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.

Abstract

Moving forward the children use a more compact method.



A common misconception:
Pupils sometimes begin subtracting with the left hand column first.

In tens and ones and other formal vertical subtraction calculations, children sometimes take the smaller digit from the larger, regardless of whether it is part of the larger or smaller number.

e.g. 945
- 237
712

Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract

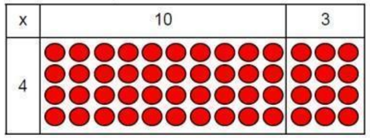
Year 3 Multiplication

Grid Method

Concrete

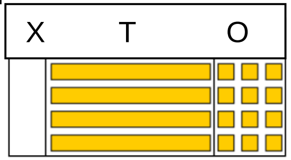
Show the link with arrays to first introduce the grid method.

4 rows of 10
4 rows of 3

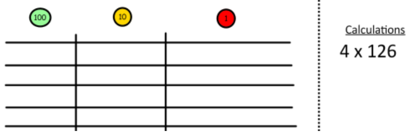


Move on to using Base 10 to move towards a more compact method.

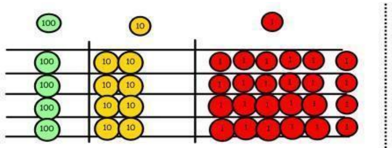
4 rows of 13



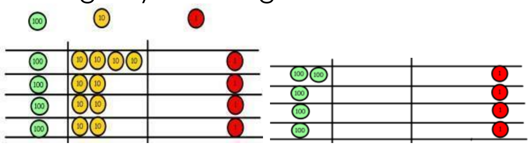
Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.



Fill each row with 126.



Add up each column, starting with the ones making any exchanges needed.

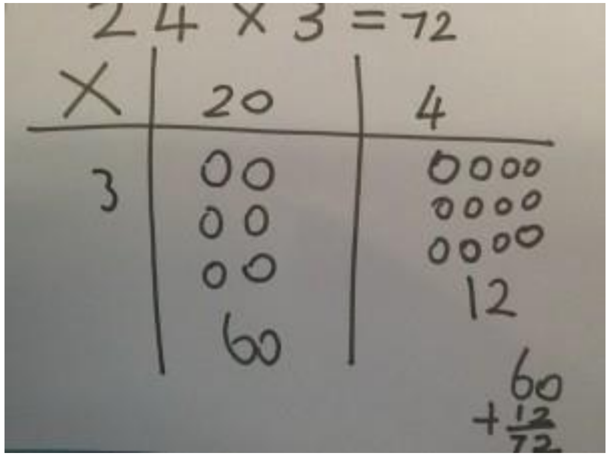


Then you have your answer.

Pictorial/Jottings

Children can represent the work they have done with place value counters in a way that they understand.

They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



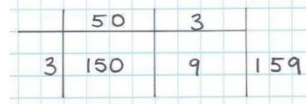
Abstract

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

x	**30**	**5**
7	**210**	**35**

210 + 35 = 245

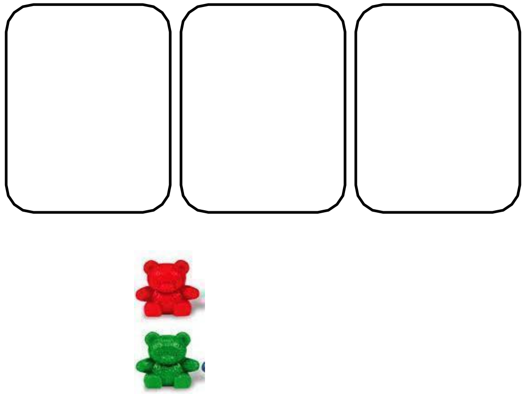
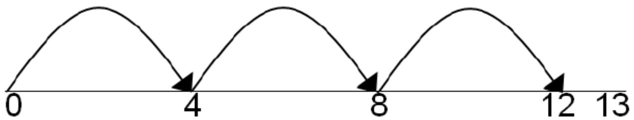
53 x 3 = 159

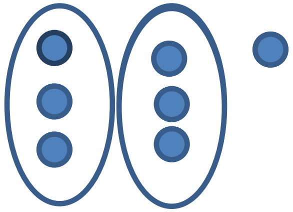
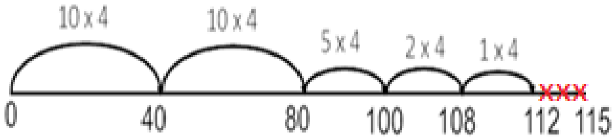


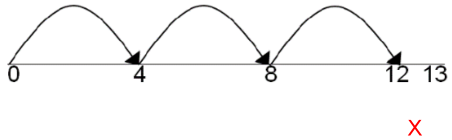
Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

A common misconception:
Place value errors when performing written calculations can cause problems for even able pupils.
Pupils do not understand that x10 and then x 10 again, is the same as x100. They prefer to learn 'add a zero' and so limit their understanding.

<table>
<tr><th colspan="2">Objectives, Strategies & Vocabulary</th><th>Concrete</th><th>Pictorial/Jottings</th><th>Abstract</th></tr>
<tr>
<td>Year 3 Division</td>
<td>Division with a remainder</td>
<td>14 ÷ 3 =
Divide objects between groups and see how much is left over
</td>
<td>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.
13÷4=


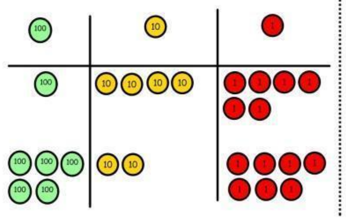
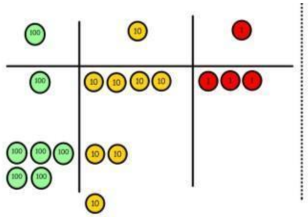
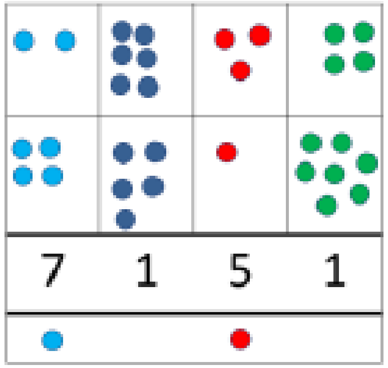
Draw dots and group them to divide an amount and clearly show a remainder.

7÷2=3r1
This can be shown on a number line.
115÷4 =


10 + 10 + 5 + 2 + 1 = 28 r 3</td>
<td>Complete written divisions and show the remainder using r.
13÷4=3r1


The remainder is shown with an ‘X’ on the line.

Using ‘What I know is needed for larger numbers:

E.g. 156 ÷ 6
I know 10 x 6 = 60 Total: 60
I know 10 x 6 = 60 120
I know 2 x 6 = 12 132
I know 2 x 6 = 12 144
I know 2 x 6 = 12 156
26</td>
</tr>
</table>

	Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract
Year 4 Addition	Column method- regrouping. (Estimate your answer before working out)	Make both numbers on a place value grid.  146 + 527 Add up the ones and exchange 10 ones For one 10  146 + 527 Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.	Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding. 	Addition using the compact method, carrying the tens underneath: 536 + 85 621 1 1 789 + 642 becomes 7 8 9 + 6 4 2 1 4 3 1 1 1 Answer: 1431 **Year 4: Column method- regrouping.** **(up to 4 digits)** Children will use addition with decimals; especially when working with money. £ 13.45 + £ 12.37 £ 25.82 1

Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract

Year 4 Subtraction

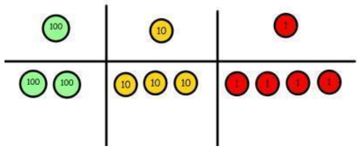
Objectives, Strategies & Vocabulary

Column method with regrouping

Concrete

Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges.

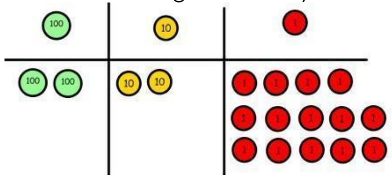
Make the larger number with the place value counters



Calculations

234
- 88

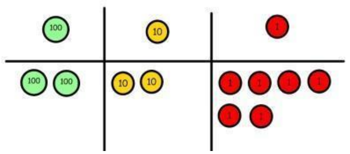
Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Calculations

234
- 88

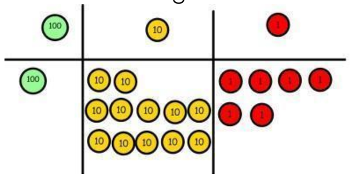
Now I can subtract my ones.



Calculations

234
- 88

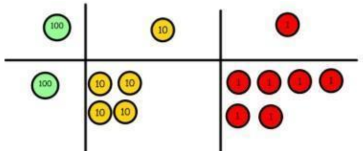
Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens.



Calculations

234
- 88

Now I can take away eight tens and complete my subtraction



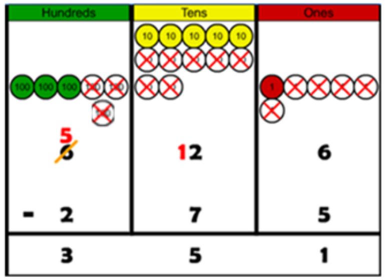
Calculations

$^{1}\cancel{2}{}^{12}\cancel{3}{}^{1}4$
- 88
146

Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.

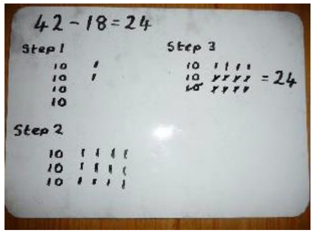
Pictorial/Jottings

Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



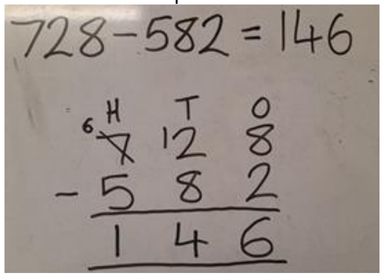
When confident, children can find their own way to record the exchange/regrouping.

Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.



Abstract

Moving forward the children use a more compact method.



874 – 523 becomes

$$\begin{array}{r} 8\ 7\ 4 \\ -\ 5\ 2\ 3 \\ \hline 3\ 5\ 1 \\ \hline \end{array}$$

Answer: 351

Children will need to use decimals when calculating; especially with money.

£17.36
£13.25
£ 4.11

Year 4: Column method with regrouping. (up to 4 digits)

$$\begin{array}{r} 2\,{}^{3}\cancel{4}\,{}^{1}5\,6 \\ -\ 1\,3\,8\,5 \\ \hline 1\,0\,7\,1 \\ \hline \end{array}$$

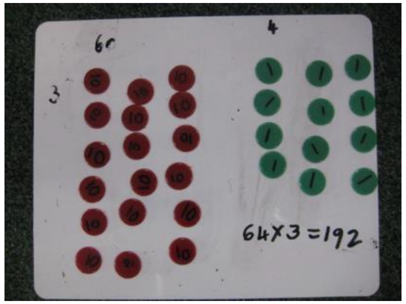
Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract

Year 4 Multiplication

Column Method

Concrete

Children can continue to be supported by place value counters at the stage of multiplication.



It is important at this stage that they always multiply the ones first and note down their answer followed by the tens which they note below.

Pictorial/Jottings

Expanded column multiplication (3d x 1d)

$$
\begin{array}{r}
327 \\
\times \quad 4 \\
\hline
28 \\
80 \\
1200 \\
\hline
1308 \\
\hline
\end{array}
$$

Abstract

Continue with Grid Method from Year 3 until ready to move on to columns.

Start with multiplying by one digit numbers and showing the clear addition alongside the grid.

×	30	5
7	210	35

210 + 35 = 245

Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

<u>Short Multiplication</u>

53 x 3 = 159

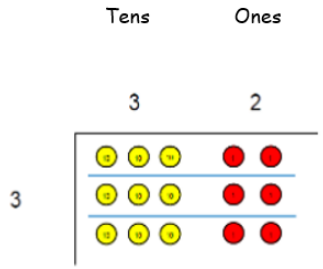
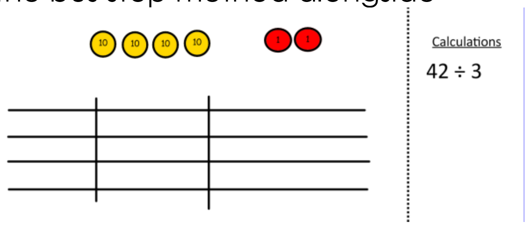
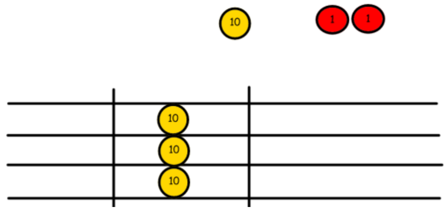
	50	3	
3	150	9	159

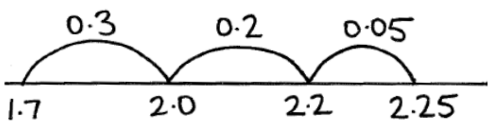
	H	T	O	
		3	4	
×			5	
	1	7	0	
	1	2		

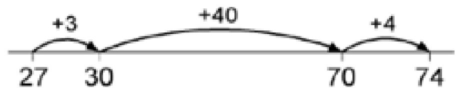
X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

	H	T	O
	2	4	5
×			4
	9	8	0
	1	2	

Year 4: 2 and 3 digit multiplied by 1 digit.

Objectives, Strategies & Vocabulary	Concrete	Pictorial/Jottings	Abstract
ear 4 Division Short division	 **96÷3=32** Use place value counters to divide using the bus stop method alongside  42 ÷ 3= Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.  We exchange this ten for ten ones and then share the ones equally among the groups. We look how much in 1 group so the answer is 14.		Begin with divisions that divide equally with no remainder. $\begin{array}{r} 2\;\;1\;\;8 \\ 4\overline{)\,8\;\;{}^{3}7\;\;2} \end{array}$ Move onto divisions with a remainder. $\begin{array}{r} 8\;\;6\;\;r\;2 \\ 5\overline{)\,4\;\;3\;\;{}^{3}2} \end{array}$ **Year 4: Short division (up to 3 digits by 1 digit)**

	Objectives, Strategies & vocabulary	Concrete	Pictorial/Jottings	Abstract
Years 5 & 6 Addition	Column method with regrouping +, add, addition, more, plus make, sum, total, altogether, score , double, near double, one more, two more… ten more… one hundred more how many more to make …? how many more is… than …? how much more is…? Carry Estimate Equals Number bonds Boundary Inverse		When adding Decimals – partition number to be added into manageable steps using number bonds to 10 and add using a number line (building on previous knowledge of adding in this way).  1.7 + 0.55 = 2.25	Year 5 - Column method- regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places) Year 6 - Column method- regrouping. (Decimals- with different amounts of decimal places) **256 + 613 + 25** 256 + 613 + 25 14 80 800 894 Once this has been mastered, pupils move onto expanded column. Once all the digits have been added, the answer should be written underneath. Emphasis should be put on the place value so that children understand what they are adding (for example the first question should be phrased as **fifty add seventy**, not five add seven). Once expanded method has been mastered, pupils move quickly onto compact method. Digits in the smallest place value column should be added first. As with other methods, emphasis should be put on the place value of the digits being used. E.g. **4626 + 1573** 4426 + 1573 5999 20 · 01 4 · 56 + 0 · 3 24 · 87 When digits are carried they should be written below the question. **7.76 + 16.43** 7.76 + 16.43 + 11 24.19 Carried digits are written here to remove the common mistake of forgetting to add them on.

	Objectives, Strategies & vocabulary	Concrete	Pictorial/Jottings	Abstract
Years 5 & 6 Subtraction	Column method with regrouping subtract, take (away), minus, leave, difference, decrease how many are left/left over? one less, two less… ten less… one hundred less how many fewer is… than …? how much less is…? difference between half, halve =, equals, sign, is the same as, exchanging, carrying, partitioning How many have gone? Fewer, difference between, missing numbers, boundary. Inverse.		Pupils should use a number line to count up from the smallest number to the largest. They should aim to get to a multiple of 10, 100 or 1000 to make the working out easier. E.g. **74 - 27**  40 + 4 + 3 = 47	Year 5 - Column method with regrouping. (with more than 4 digits) (Decimals - with the same amount of decimal places) Year 6 - Column method with regrouping. (Decimals - with different amounts of decimal places) Pupils should write the first number un-partitioned and then write the number to be subtracted underneath, making sure to keep the place value columns correct. Borrowing should be done in the same way as expanded columns. The digits will still be referred to using their correct place value, so in the example below you would move **ten** out of the **eighty** to make it **seventy**, the ten moves to join the two to give **twelve**. E.g. **582 - 455** $5\,{}^{7}8\,{}^{1}2$ $-\ 4\ 5\ 5$ $1\ 2\ 7$ *When pupils have reached this point they should be able to subtract decimals (including those with a mixed number of digits) as well as more than one number.*

<table>
<tr><th colspan="2">Objectives, Strategies & vocabulary</th><th>Concrete</th><th>Pictorial/Jottings</th><th>Abstract</th></tr>
<tr>
<td>Years 5 & 6 Multiplication</td>
<td>Column method (long)

lots of, groups of
x, times,
multiplication,
multiply,
multiplied by
multiple of, product
once, twice, three times,
four times, five times… ten times…
times as (big, long, wide and so on)
repeated addition
array
row, column
double,
exchanges
factor
product
halving, doubling
number patterns
multiplication table
inverse
square, cube</td>
<td></td>
<td></td>
<td>Year 5 – Column Multiplication (multi digit up to 4 digits by a 2 digit number)
Year 6 – Column multiplication (multi digit up to 4 digits by a 2 digit number)
Once pupils have a firm understanding of addition using formal columns, including carrying digits into the next column, they should begin to use short multiplication to multiply by 1 digit. The smallest units should always be multiplied first. Where an answer to a multiplication is a 2 digit answer, the largest digit should be ‘carried’ into the next column and written in a smaller font below the question, where it can be added on.
<table>
<tr><td></td><td>H</td><td>T</td><td>O</td><td></td></tr>
<tr><td></td><td></td><td>3</td><td>4</td><td></td></tr>
<tr><td>×</td><td></td><td></td><td>5</td><td></td></tr>
<tr><td></td><td>1</td><td>7</td><td>0</td><td></td></tr>
<tr><td></td><td>1</td><td>2</td><td></td><td></td></tr>
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<tr><td></td><td>2</td><td>4</td><td>5</td></tr>
<tr><td>×</td><td></td><td></td><td>4</td></tr>
<tr><td></td><td>9</td><td>8</td><td>0</td></tr>
<tr><td></td><td>1</td><td>2</td><td></td></tr>
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The calculations needed can be written next to the sum to aid pupils if needed.

At this stage, grid method should continue to be used for TO x TO or higher.

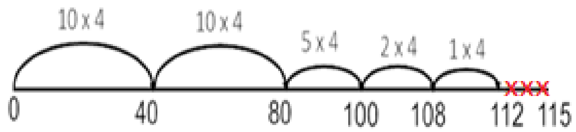
When pupils have a secure knowledge of short multiplication by 1 digit and multiplying mentally by 10, 100 and 1000 they should begin using long multiplication for multiplying 2 or more digits.

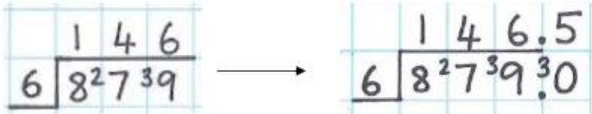
	H	T	O
		2	2
×		3	1
		2	2
	6	6	0
	6	8	2

Th	H	T	O
	2	3	4
×		3	2
	4	6	8
$_{1}7$	$_{1}0$	2	0
7	4	8	8

Pupils can still write the questions out if needed for understanding when working with this method.

If students forget they are multiplying by a 10 we can box the zero in (below) to ensure there is always a zero in this position – understanding of place value does need to be emphasised at this point.

Objectives, Strategies & vocabulary	Concrete	Pictorial/Jottings	Abstract
Years 5 & 6 Division **Bus stop (with remainders)**		**This can be shown on a number line if not ready for the bus stop method.** **115÷4 =**  10 + 10 + 5 + 2 + 1 = 28 r 3	Year 5 - Short division (up to 4 digits by a 1 digit number Interpret remainders appropriately for the context) Year 6 - Short division Long division (up to 4 digits by a 2 digit number-interpret remainders as whole numbers, fractions or round) Pupils will be expected initially to use **short division**. Pupils will begin by dividing the highest digit in the large number by the divisor. When the final answer is achieved, any remainders should be written after the answer. Pupils should start by dividing numbers with no remainders. **362 ÷ 7 =** **5 1 r5** **7 | 3 6 [1]2** **362 ÷ 7 = 51 r5** *Pupils will also be expected to use the short division method to divide by 2-digit numbers. In these cases, pupils will write out the first five times table of the divisor (can use a partitioning method).* E.g. **3822 ÷ 12** 3 1 8 r 6 1 2) 3 8 [2]2 [10]2

	Objectives, Strategies & vocabulary	Concrete	Pictorial/Jottings	Abstract
Years 5 & 6 Division	**Bus stop method (with remainders) (Long)**		Short and long division can be used to divide decimal numbers as well; children simply need to remember to put the decimal point in exactly the same position on the answer line as it is in the question.	Children should have an understanding of how to change remainders into fractions. In this example: 19 ÷ 6 = 3 r1 the remainder can be turned into a fraction by continuing to divide it by 6. 19 ÷ 6 = 3 1/6 Children can also express a remainder as a decimal. When using either short or long multiplication, by adding a decimal point and a zero to the number being divided, we are able to carry on the calculation.  They must also remember to add a decimal point to the answer line, in the same position as the one in the question. It might be that the children will be presented with an example where they need to add more than one zero on to the number being divided.